# Spline Surface 

Splines \& Surfaces

User's guide

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### 1.1. Installation

In order to run this application, you have to use at least Windows XP. You also need both Visual C++ Redistributable for Visual Studio 2012 Update 4 package and Microsoft .NET Framework 4 Client Profile installed. Both can be downloaded from [12] and [13] or you can find them on the included CD. You have to choose correct version for $x 86$ or $x 64$ architecture (32-bit and 64-bit).

File "muparserd32.dll" and/or "muparserd64.dll" have to be in same folder as executable file "Splines and surfaces.exe" in order to secure full functionality.

### 1.2. Graphic user interface



Figure 0-1 Main window of Splines \& Surfaces
When you launch Splines \& Surfaces application, main window will show up (Figure 0-1). Menu strip is located in its top part. Tool strip is just under menu. The largest part of window is occupied by $3 D$ window. All graphic objects you create and modify will be displayed here. On the right side of $3 D$ window are Auxiliary window and Control box under it.

Inside menu under File tab, you can choose

- New create new object
- Save save currently modelled object
- Load load any previously saved object.

If you choose Save or Load, you will be prompted to choose folder and/or file name. Last option in this submenu is Quit. Under second tab Options you can find Display options and Curve and patch options. Display options are described below.

Curve and patch options include functions:

- Degree reduction for Bézier curve or patch

Reduces degree of Bézier curve or patch with preselected method and recomputes current object. In case of patch, degree reduction will happen in selected parametric direction. If you hover mouse over this option, 3 more options will show up. These define method for reducing degree. You can choose one of them method by clicking on it.

- Method $i / n$
- Method 0:1
- Method sum
- Degree elevation for Bézier curve or patch

Elevates degree of Bézier curve or patch with preselected method and recomputes current object.

- Reset weights of control points

Sets weights for all control points to 1.

- Reset multiplicity of control points

Sets multiplicity for all control points to 1 .

- Set open knot vector

Sets knot vector type for selected parametric direction to open.

- Set uniform knot vector

Sets knot vector type for selected parametric direction to uniform.

- Set spline type

Sets spline type to one of following:

- Default - no end condition
- Double endpoints
- Triple endpoints
- Relaxed spline
- Clamped spline
- Cyclic spline
- Acyclic spline
- Quadratic condition

Under Arcs tab you can choose:

- Parametric curve
- Bézier arc
- Integral
- Rational
- Hermite arc
- Cubic
- Quintic

Under Splines tab you can choose:

- Hermite spline
- Cardinal spline
- Bézier spline
- Integral approximation Bézier spline
- Rational approximation Bézier spline
- Integral interpolation Bézier spline
- Rational interpolation Bézier spline
- Beta spline
- B-spline:
- Integral
- Rational - NURBS

Under Surfaces tab you can choose:

- Parametric surface
- Affine surface
- Surface of revolution
- Extruded surface
- Coons patch:
- Linear
- Bilinear
- Partially bicubic
- Bicubic
- Fergusson patch
- Adini twist
- Custom twist
- Bézier patch:
- Integral
- Rational
- B-spline patch:
- Integral
- Rational - NURBS

Under About you can find information about author and version of software.
Just below Menu strip you see toolbar. It contains icons representing functions:

- New
- Save
- Load
- Reset view
- Top view
- Degree elevation for Bézier curve or patch
- Degree reduction for Bézier curve or patch
- Set open knot vector
- Set uniform knot vector
- Other buttons include all types of arcs, curves and surfaces. If you hover mouse cursor over any of them, tool tip shows up with information about object type.

On the right side of window, you can see large white square. This is Auxiliary window. Here, blending functions are displayed for most objects and also knot vector if it is used. In that case you can press right mouse button to show Knot vector options window. We describe it in detail later. Control box is located under this white square (described below).

### 1.3. Display options



Figure 0-2 Display options
Display options window (Figure 0-2) covers all options which can modify how objects you create will look. You can change size of points, width of lines, curve and lines for wireframe. Next two numeric boxes control how precise will be object enumerated. It defines number of points that will form the object. Setting this value too high may result in lag, especially for parametric curve and surfaces. Last numeric box enables you to change size of grid in a plane $z=0$.

You can choose secondary colour for spline. Each odd segments will be coloured with this colour, so you can clearly see end points of each segment. Surface colour defines its base colour, which can be modified by shading.

Application offers you choice between orthographic and perspective projection. You can change it at any time. If you want display points as squares and not circles, deselect point smoothing option. Line smoothing applies antialiasing to lines and makes them little bit wider.

Show / hide section lets you select which objects will be displayed. Vertices and Control polygon / net applies for all curves but only for tensor product surfaces. Grid, Axes, Isocurves and Point on object are self-explanatory. Illustration points are points, which will show you process of curve enumeration. This applies in the case of Bézier arc, Bézier spline, B-spline and rational curves of these types. For (rational) Bézier approximation spline, control points for each segment and for interpolation spline points required to create $C^{2}$ continuous spline are displayed. If you
have Hermite spline modelled then it will show you tangents at control points．If you hide control polygon，tangents are still displayed as points and lines illustrating algorithms are still visible．

In Surface display mode you can choose，how the surface will look．Default option is Wireframe，which is formed by lines connecting enumerated points on surface．Faceted surface displays whole patch with single colour．Shaded surface and Shaded surface with reflections incorporate lights into the scene．Last option Point cluster renders only points on surface．

## 1．4．Control box

| Parameter |  | Select to |  | Row or column |
| :---: | :---: | :---: | :---: | :---: |
| （0）U 0，50 ${ }_{-}^{\text {－}}$ |  | （－change $u / v$ |  | 0 聍 |
| $\bigcirc \mathrm{V} 0.5$ | $50 \stackrel{-1}{*}$ | －change kn |  |  |
| Modifiers |  |  |  |  |
| 3 会 | Degree U | 3 － | Degree V |  |
| 1 考 | Multiplicity | 1，00 | Weight |  |
| 1.0 － | Beta 1 and s $0.1 \leqslant$－${ }^{\text {a }}$ |  |  |  |
| u interval 0，0；1，0 |  |  | End condition |  |
| $v$ interval | 0，0；1，0 |  | Default |  |
|  |  |  | Double endpoints |  |
| Rotation | 0； 180 |  | Triple endpoirRelaxed |  |
|  | 1：1：1 |  | Clamped Cyclic |  |
|  |  |  | Acyclic |  |
| Affine surface from curve |  | Evaluate text boxes | Quadratic condition |  |
| $\mathrm{CO}(\mathrm{u}) / \mathrm{p}(\mathrm{u}) \mathrm{u}: \mathrm{u}^{\wedge} 2 ; 1-\mathrm{u}$ |  |  |  |  |
| C1（u） | $u+1 ; u^{\wedge} 2-1 ; 2-u$ |  |  |  |
| DO（v） | v：$-v^{\wedge} 2 ; v+1$ |  |  |  |
| D1（v） | 1＋v：1－v＾2；v |  |  |  |
| $\mathrm{x}(\mathrm{u}, \mathrm{v})$ | $3 *\left(-1+2^{*} \mathrm{U}\right)$ |  |  |  |
| y（u，v） | $3^{*}\left(-1+2^{*} \mathrm{v}\right)$ |  |  |  |
| $z(u, v)$ | $\sin (u)+\cos (\mathrm{v})$ |  |  |  |
| Control net |  |  |  |  |
| Plane | 4 － | \＃of points U | Scale |  |
| Random | $4 \stackrel{\mid-1}{*}$ | \＃of points V | 3 － | $\stackrel{1}{\square}$ |

Figure 0－3 Control box
Control box（Figure 0－3）offers functionality to modify objects and each sub－section has tool tips ready to display．Under Parameter you can choose parameter，for which blending functions will be displayed in Auxiliary window．You can change value of highlight parameter by setting it inside numeric box or if change $u / v$ is selected，by left clicking inside Auxiliary window．If change knots is selected and knots are displayed inside Auxiliary window（blue hollow rectangles），you can select one of them by pressing left mouse button，if mouse cursor is close enough．Selected knot will then become blue filled rectangle．By next left mouse click inside this window，you can define new value for selected knot and it will become hollow rectangle again （unselected）．

Numeric box Row or column specifies which row or column of blending functions will be displayed．In $u$ and $v$ parametric direction it shows selected row and column，respectively． This option applies only for rational Bézier and B－spline surfaces．

Modifiers group include numeric boxes for degree in both parametric directions, multiplicity, weight and $\beta_{1}, s$ and $\beta_{2}$ parameters. Values of multiplicity and weight are used in procedure of modifying control point.

Inside text boxes you can define curves, which will be used to construct surfaces. Text boxes labelled $u$ interval, $v$ interval and Rotation interval define intervals for $u, v$ and rotation, respectively. Content of these text boxes have to be two numbers separated by ";" and you can use "," as decimal separator. You can define these numbers as value of a function, for example $\sin (5 * \pi)$. Axis text box defines axis of rotation for surface of revolution. Four curves (first of them serves also for parametric curve) can be defined via text boxes labelled $C O(u) / p(u), C l(u)$, $D O(v)$ and $D 1(v)$. These five text boxes therefore need three numbers or expressions, respectively. Next three text boxes $x(u, v), y(u, v)$ and $z(u, v)$ are used to define parametric surface. Here only one expression per text box is needed. In all text boxes use only the parameter denoted in parenthesis for each respective expression. You can use any combination of addition, subtraction, multiplication, division of numbers or functions, all of which have predefined keyword. They are listed in table of keywords on the page 12.

If you want to save (and later load) your object created from text boxes' definition, you have to make sure that all expressions do not have spaces in them, but are separated with at least one space. For example (good)

- $u+1 ; u^{\wedge} 2-2 ; \sin (u)+1,5$ for text boxes $C 0(u) / p(u), C l(u), D 0(v)$ and $D l(v)$
- $u * 2+1$ for text boxes $x(u, v), y(u, v)$ and $z(u, v)$
- 0,$1 ; 1,3$ for intervals text boxes
as opposed to (wrong)
- $u+1 ; u^{\wedge} 2-2 ; \sin (u)+1$ for text boxes $C 0(u) / p(u), C 1(u), D 0(v)$ and $D 1(v)$
- $u * 3,8+1.5$ for text boxes $x(u, v), y(u, v)$ and $z(u, v)$
- 0,$0 ; 1,3$ for intervals text boxes
which will not cause an error during evaluating but loading object will fail completely. You can use constants $\pi$ and $e$, which have keywords $p i$ and $e$, respectively .

Evaluate text boxes button serves to refresh currently modelled object. If you check Affine surface from curve checkbox, then affine surface will be constructed from curve, you have previously modelled.

You can choose End condition from list. In the case it is not supported by current spline, you will see error message. In the case of Clamped application's input mode will change to awaiting coordinates for tangent vectors. You have to define 2 tangent vectors. Process of defining these tangents is totally same as defining points.

Control net group defines parameters for initialisation of control net for tensor product surfaces and for bicubic Coons patch. You can change number of points via two numeric boxes (only for tensor product surfaces) and scale affects overall size of control net. List box contains options:

- Plane: control points have $z$ coordinate 0
- Elevated: control points are located on an arc
- Random: control points have random z coordinate


### 1.5. Controls

When the application starts, it is awaiting coordinates of points. If you move mouse cursor, lines indicating actual cursor coordinates will show up. You can press left mouse button to set $x$ and $y$ coordinates of control point. Now application awaits $z$ coordinate. You can either press left mouse button, which will set $z$ coordinate to cursor position or press right mouse button to set $z$ coordinate to 0 . Application awaits again $x$ and $y$ coordinates for next control point. You can add as many points as you need. If you are finished with defining control points, you press right mouse button to cancel mode of defining new control points. In this mode coordinates of cursor are not displayed.

If you want to change position of any control point, you move mouse cursor above it. It will become selected if the cursor is close enough and application will let you know by highlighting that point with red colour and drawing lines to better illustrate its coordinates. Now press left mouse button. Again, define new $x$ and $y$ coordinates by left clicking. If you press right mouse button, $x$ and $y$ coordinates will not change. After pressing one or the other, you have to set $z$ coordinate. You can set it to mouse cursor position by pressing left mouse button or to 0 by pressing right mouse button. It is essentially the same procedure as defining new point.

If you are not in point defining mode and no control point is selected, you can press left mouse button to start rotating $3 D$ window. Moving mouse cursor now rotates viewport. You can accept new view by pressing left mouse button or right mouse button. Zooming in and out is done via turning the mouse wheel at any time. If you press arrows on keyboard at any time with mouse cursor inside $3 D$ window, view port will shift in that direction.

If you are not in point defining mode, you can press right mouse button to show Context тепи. (Figure 0-4)

```
Add control vertex at the end
Add control vertex at the beginning
Specify vertex
Delete control vertex
Change multiplicity of control vertex
Change weight of control vertex
Show multiplicity of control vertex
Show weight of control vertex
Cancel
```

Figure 0-4 Context menu

It offers you choices of:

- Add control vertex at the end

Enables you to add more control points to end of control points' sequence.

- Add control vertices at the beginning

Enables you to add more control points to beginning of control points' sequence.

- Specify vertex

Show form for specifying vertex. It is described in detail below.

- Delete control vertex

If you have selected control point, it will delete it form sequence of control points.

- Change multiplicity of control vertex

Sets multiplicity of selected control vertex to value specified in numeric box labelled Multiplicity located inside Control box

- Change weight of control vertex

Sets weight of selected control vertex to value specified in numeric box labelled Weight located inside Control box

- Show multiplicity of control vertex

Displays message window with value of multiplicity for selected vertex.

- Show weight of control vertex
- Displays message window with value of weight for selected vertex.
- Cancel

Closes this context menu.
Choosing Specify vertex option will open new window. (Figure 0-5) If offers you two or only one choice, according to whether you have or have not selected control point (or tangent) before.


Figure 0-5 Specify vertex
In the first case, you can only add new vertex to the end of control points' sequence. In the second case you can also modify selected vertex. Its actual coordinates are displayed too so you can see them at once. Check boxes allows you to change only some attributes of selected point. They do not have effect on attributes of new point.

### 1.6. Modelling objects

Objects are divided to two main groups, splines and surfaces. For splines, except one curve type, you have define control polygon by yourself. In the case of surfaces, application first creates the object and then you can modify it.

For arcs and spline, you simply create sequence of control points and then choose type of curve you want. You can modify it by changing position, weight and multiplicity of control vertices or tangents (in some cases even vectors of second derivative), modifying knot vector,
changing end condition or using numeric boxes in control box if current spline is modifiable by them. If you select Clamped, application awaits coordinates for two tangent vectors. You define them exactly as control points. Changes will occur immediately.

One exception is parametric curve. This is defined by text box inside Control box and this is the only way to modify its shape (other than changing its enumerating interval). When you modify this expression, either choose Parametric curve from menu, click on its icon in toolbar or click on Evaluate text boxes button.

For surfaces, there are generally two ways to model them. For Bézier, B-spline and Bicubic Coons patch, control net is created by application. Scale of this control net is controlled by Size numeric box inside Control net in Control box. \# of points $U$ and\# of points $V$ do not affect bicubic Coons patch. You can modify it by changing position or weights, modifying knot vector and choosing different types of twist vectors.

If you model rational Bézier or B-spline surface, you can select which row or column of blending functions you want to see in Auxialiary window. You can select it in numeric box Row or column.

Other group consists of affine surfaces, Coons patches other than bicubic and parametric surface. All of them are defined by multiple text boxes inside Control box. Here you can change predefined expressions to modify shape of surface. In order to change take effect, you either choose desired type of surface in menu, click on its icon in toolbar or if you have any one of these surfaces already created, click on Evaluate text boxes button.

Affine surfaces offers you option to create surface of revolution or extruded surface either from curve, also called profile curve, defined in text box or the one you have modelled. First, you have to create a sequence of control points and select type of curve. Then you have to choose either type of affine surface and then select Affine surface from curve option. You can uncheck it at any time. To modify curve, which is now used to create affine surface, click on any one type of curve. This will bring you to curve modelling mode, where you can modify curve's shape. When you are satisfied, you can again select Affine surface from curve option.

If you will change precision with this option active, application will show message saying that no curve is defined. All you have to do is to select type of curve you want to be profile curve, then select type of affine surface and then finally select Affine surface from curve option.


Figure 0-6 Knot vector options
Application will show you this window once you have B-spline curve or B-spline patch modelled and you press right mouse button while mouse cursor is in Auxiliary window. (Figure 0-6) In the case of patches, you can choose knot vector for respective parametric direction. Two numeric boxes change index and value of knot. If you change their value, labels for operations will change itself so you can clearly see which knot with its current value will be modified to new value, its multiplicity will be increased or which knot value will be inserted or removed from knot vector. Other two operations serves purpose of setting knot vector type to open or uniform. If you have clicked on Apply button, move mouse cursor away from it and new knot vector will be written down.

Table 1 Keywords

| KEYWORD | \# OF ARGUMENTS | FUNCTION |
| :---: | :---: | :---: |
| sin | 1 | sine function |
| cos | 1 | cosine function |
| tan | 1 | tangens function |
| asin | 1 | arcus sine function |
| acos | 1 | arcus cosine function |
| atan | 1 | arcus tangens function |
| sinh | 1 | hyperbolic sine function |
| cosh | 1 | hyperbolic cosine |
| tanh | 1 | hyperbolic tangens function |
| asinh | 1 | hyperbolic arcus sine function |
| acosh | 1 | hyperbolic arcus tangens function |
| atanh | 1 | hyperbolic arcur tangens function |
| ${ }^{\text {log2 }}$ | 1 | logarithm to the base 2 |
| $\log 10$ | 1 | logarithm to the base 10 |
| 109 | 1 | logarithm to the base 10 |
| 1 n | 1 | logarithm to base e (2.71828...) |
| exp | 1 | e raised to the power of $x$ |
| sqrt | 1 | square root of a value |
| sign | 1 | sign function -1 if $\mathrm{x}<0 ; 1$ if $x>0$ |
| rint | 1 | round to nearest integer |
| abs | 1 | absolute value |
| min | var. | min of all arguments |
| max | var. | max of all arguments |
| sum | var. | sum of all arguments |
| avg | var. | mean value of all arguments |

